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A GENERAL PREWHITENING PROCEDURE FOR PROCESS AND MEASUREMENT NOISES

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In data reconciliation and gross error detection as in quality control charts most methods assume that the data are serially independent. This assumption is convenient for mathematical treatment, but is often contradicted by experimental evidence. Recent work (Kao, *et al.*, 1990, 1991) examines alternative procedures for mitigating such effects. One of these procedures is to prewhiten the process data before applying the usual data treatment methods. Prewhitening may be carried out using the Bryson and Henrikson method (1968), which is applicable to first order autoregressive models. In this note we propose a prewhitening procedure which is applicable to any autoregressive moving average (ARMA) model, and which causes only a modest increase in the state space dimensions.
KEYWORDS White noise Prewhittening Serial correlation.

INTRODUCTION

In data reconciliation and gross error detection as in quality control charts most methods assume that the data are serially independent; see Tamhane and Mah (1985) and Mah (1990) for a review of these methods. This is convenient for mathematical treatment but is often contradicted by the experimental evidence. Process data may be correlated due to many reasons, e.g., process dead time, feedback control, process dynamics, model misspecification, residual measurement errors etc.

The effect of serial correlations of process measurements in the steady state chemical process has been studied by Kao *et al.* (1990). We showed the extreme sensitivity of the measurement test for gross error detection to serial correlations and pointed out that serial correlations should not be ignored. We also proposed a prewhitening procedure to filter out the serial correlations resulting in prewhitened (uncorrelated) residuals to which standard techniques of gross error detection for independent process data can be applied. (Note that independence implies no serial correlation; the converse is also true for normally distributed data.)

However, in reality even for steady state operations, process conditions vary continually about a nominal steady state. A truly steady state is almost never attained. A dynamic system will be a better representation of a real process. A possible way to account for these problems is using the Kalman filter algorithm via the state space model (Bellingham and Lees, 1977; Watanabe and Him-

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melblau, 1982; Narasimhan and Mah, 1988). A major drawback of these papers is that they assume that the process and measurement noises are white.

Bryson and Henrikson (1968) proposed an approach based on measurement differencing for the estimation of state variables in a stochastic linear dynamic system subject to serially correlated measurement noise. They considered the autoregressive model of order one (AR(1)) for the measurement noise, in which case their approach is equivalent to prewhitening, i.e., filtering out serial correlations so as to make the noise white. However, by keeping the state space dimension unchanged their approach introduces cross-correlation between the process noise and the prewhitened measurement noise, which must be removed by making a second transformation. These two transformations lead to a model in the standard Kalman filter form which allows the usual optimal state variable estimation methods to be employed. Their approach avoids the increase in the dimensionality of the state space and the ill-conditioned computations associated with the augmented state approach suggested by Kalman (1963).

It should be pointed out that the aforementioned advantages of the Bryson and Henrikson's approach hold only for the AR(1) model. For more general serial correlations, e.g., the noise following an autoregressive moving average (ARMA) model of general order proposed by Box and Jenkins (1976), their approach loses the advantage mentioned above and leads to some additional complications. We propose a general prewhitening approach, which works for any ARMA model. This approach does involve an increase in the dimensionality of the state space, but not to the extent of making the computations ill-conditioned or infeasible.

STATEMENT OF THE PROBLEM

Consider a linear dynamic discrete-time system described by
State Equation:

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{w}(t) \tag{1}$$

Measurement Equation:

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{v}(t) \tag{2}$$

where

$\mathbf{x}(t)$ is an $n \times 1$ vector of state variables at time t
$\mathbf{u}(t)$ is an $r \times 1$ vector of control input variables at time t
$\mathbf{w}(t)$ is an $n \times 1$ vector of process noises at time t
$\mathbf{y}(t)$ is an $m \times 1$ vector of output variables (measurements) at time t
$\mathbf{v}(t)$ is an $m \times 1$ vector of measurement noises at time t
$\mathbf{A}$ is an $n \times n$ state transition matrix
$\mathbf{B}$ is an $n \times r$ input matrix

and

$\mathbf{H}$ is an $m \times n$ measurement matrix.

The noises $\mathbf{w}(t)$ and $\mathbf{v}(t)$ are assumed to be Gaussian with mean $\mathbf{0}$ and covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively.

It is well known that the Kalman filter provides a minimum variance unbiased state estimate of the discrete-time linear dynamic system described by Eqs. (1) and (2), where the noises $\mathbf{w}(t)$ and $\mathbf{v}(t)$ are white and Gaussian. However, in actuality, the process and measurement noises may be serially correlated, i.e., the process noise sequence $\{\mathbf{w}(t), t=1, 2, \dots\}$ and measurement noise sequence $v(t), t=1, 2, \dots\}$ in Eqs. (1) and (2) may not be white and may follow the ARMA(p, q) models which are defined by

$$[\mathbf{I}-\Phi(z)]\mathbf{v}(t)=[\mathbf{I}-\Theta(z)]\mathbf{a}(t)$$

or

$$\mathbf{v}(t)-\phi_1\mathbf{v}(t-1)-\cdots-\phi_p\mathbf{v}(t-p)=\mathbf{a}(t)-\theta_1\mathbf{a}(t-1)-\cdots-\theta_q\mathbf{a}(t-q) \quad (4)$$

where

$$\Phi(z)=\phi_1 z^{-1}+\phi_2 z^{-2}+\cdots+\phi_p z^{-p} \quad (5)$$

$$\Theta(z)=\theta_1 z^{-1}+\theta_2 z^{-2}+\cdots+\theta_q z^{-q} \quad (6)$$

$\phi_1, \dots, \phi_p, \theta_1, \dots, \theta_q$ are constant, known matrices. $\mathbf{I}$ is the identity matrix of order m and z^{-1} is the backshift operator, i.e., $z^{-j}\mathbf{v}(t)=\mathbf{v}(t-j)$ for $j \geq 1$. Similarly, an analogous relationship between $\mathbf{w}(t)$ and $\mathbf{n}(t)$ can be formulated with different values of p and q in general. In the above derivation $\{\mathbf{a}(t), t=1, t=2, \dots\}$ and $\{\mathbf{n}(t), t=1, 2, \dots\}$ are white noise sequences with zero mean and constant covariance matrices $\mathbf{Q}^*$ and $\mathbf{R}^*$, respectively. (Note that $\mathbf{Q}=\mathbf{Q}^*$ and $\mathbf{R}=\mathbf{R}^*$ if $\mathbf{w}(t)$ and $\mathbf{v}(t)$ are white, respectively.)

We wish to obtain a minimum variance unbiased estimate of the state variable $\mathbf{x}(t)$ based on measurements up to and including $\mathbf{y}(t)$.

METHODS FOR DEALING WITH SERIALLY CORRELATED MEASUREMENT NOISE

Measurement Differencing Approach

First let us consider the case where the process noise is white, but the measurement noise is serially correlated and follows the ARMA(p, q) model. Bryson and Henrikson (1968) considered a special case of Eq. (4) with $p=1$, $q=0$, i.e., an AR(1) model for measurement noise, and the system described by Eqs. (1) and (2) with $\mathbf{Bu}(t)=0$. Then by defining a new measurement variable at time $t+1$ by

$$\mathbf{y}'(t)=\mathbf{y}(t+1)-\phi_1\mathbf{y}(t)=(\mathbf{HA}-\phi_1\mathbf{H})\mathbf{x}(t)+[\mathbf{Hw}(t)+\mathbf{a}(t+1)]=\mathbf{H}^*\mathbf{x}(t)+\mathbf{v}'(t) \quad (7)$$

it is seen that the new measurement noise $\mathbf{v}'(t)=\mathbf{Hw}(t)+\mathbf{a}(t+1)$ forms a white noise sequence with zero mean and covariance matrix

$$E[\mathbf{v}'(t)(\mathbf{v}'(t))^T]=\mathbf{HQH}^T+\mathbf{R}^*=\mathbf{M} \quad \text{(say)} \quad (8)$$

However, $\mathbf{v}'(t)$ is cross-correlated with $\mathbf{w}(t)$, the covariance matrix between them being given by

$$E[\mathbf{w}(t)(\mathbf{v}'(t))^T] = \mathbf{Q}\mathbf{H}^T = \mathbf{N} \quad \text{(say)} \tag{9}$$

To eliminate this cross-correlation, Bryson and Ho (1969) introduced a matrix $\mathbf{D}$ with

$$\mathbf{D} = \mathbf{N}\mathbf{M}^{-1} \tag{10}$$

The state equation (1) can be rewritten by adjoining Eq. (7) as

$$\begin{aligned} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{w}(t) + \mathbf{D}[\mathbf{y}'(t) - \mathbf{H}^*\mathbf{x}(t) - \mathbf{v}'(t)] \\ &= [\mathbf{A} - \mathbf{D}\mathbf{H}^*]\mathbf{x}(t) + \mathbf{D}\mathbf{y}'(t) + [\mathbf{w}(t) - \mathbf{D}\mathbf{v}'(t)] \\ &= \mathbf{A}^*\mathbf{x}(t) + \mathbf{D}\mathbf{y}'(t) + \mathbf{w}'(t) \end{aligned} \tag{11}$$

Note that $\mathbf{y}'(t) - \mathbf{H}^*\mathbf{x}(t) - \mathbf{v}'(t) = 0$. We see that the new process noise $\mathbf{w}'(t) = \mathbf{w}(t) - \mathbf{D}\mathbf{v}'(t)$ and the measurement noise $\mathbf{v}'(t)$ are independent and white. Thus Eqs. (7) and (11) together are in the standard Kalman filter form, and hence the equations for the optimal filter, variance, gain, etc., can be developed in a straightforward manner. In doing this, note that the dimension of the state space has not changed. However, if this approach is applied to a higher order ($p \geq 2$, $q \geq 1$) ARMA model, this advantage is lost and some additional complications are encountered. As an example, suppose that $\{\mathbf{v}(t),\ t = 1, 2, \ldots\}$ follows an AR(2) model. To eliminate the serial correlation of the measurement noise, we first define a new measurement at time $t+2$ according to the Bryson–Henrikson approach by

$$\begin{aligned} \mathbf{y}'(t) &= \mathbf{y}(t+2) - \phi_1\mathbf{y}(t+1) - \phi_2\mathbf{y}(t) \\ &= (\mathbf{H}\mathbf{A}^2 - \phi_1\mathbf{H}\mathbf{A} - \phi_2\mathbf{H})\mathbf{x}(t) + [(\mathbf{H}\mathbf{A} - \phi_1\mathbf{H})\mathbf{w}(t) + \mathbf{H}\mathbf{w}(t+1) + \mathbf{a}(t+2)] \\ &= \mathbf{H}^*\mathbf{x}(t) + \mathbf{v}'(t) \end{aligned} \tag{12}$$

We can see that the state space remains unchanged. However, the new measurement noise $\mathbf{v}'(t)$ at time $t+2$ is cross-correlated with $\mathbf{w}(t+1)$ as well as $\mathbf{w}(t)$. Additional transformations will be necessary to remove these cross-correlations. In general, for an AR(p) model, the new measurement noise $\mathbf{v}'(t)$ at time $t+p$ will be cross-correlated with $\mathbf{w}(t+j)$, for $j = 0, 1, 2, \ldots, p-1$. The situation becomes even more complicated for an MA(q) model. Consider a simple case in which $\{\mathbf{v}(t), t = 1, 2, \ldots\}$ follows an MA(1) model. In order to eliminate the serial correlation of the measurement noise and keep the state space unchanged, we have to define a new measurement at time $t+k$ such that

$$\mathbf{y}'(t) = \mathbf{y}(t+k) + \theta_1\mathbf{y}(t+k-1) + \cdots + \theta_1^{k-1}\mathbf{y}(t+1) + \theta_1^k\mathbf{y}(t) \tag{13}$$

with $k \to \infty$ (since $(\mathbf{I} - \theta_1 Z^{-1})^{-1} = \mathbf{I} + \theta_1 Z^{-1} + \theta_1^2 Z^{-2} + \cdots$). This approach becomes theoretically untenable since it involves infinitely many past values of measurement as well as cross-correlations between the measurement noise and process noise.

A General Prewhitening Approach

To prewhiten the ARMA(p, q) measurement noise in Eq. (2), we define a new measurement $\mathbf{y}^*(t)$ at time t such that

$$\begin{aligned}\mathbf{y}^*(t) &= [\mathbf{I} - \Theta(z)]^{-1}[\mathbf{I} - \Phi(z)]\mathbf{y}(t) \\ &= [\mathbf{I} - \Theta(z)]^{-1}[\mathbf{I} - \Phi(z)]\mathbf{H}\mathbf{x}(t) + [\mathbf{I} - \Theta(z)]^{-1}[\mathbf{I} - \Phi(z)]\mathbf{v}(t) \\ &= [\mathbf{I} - \Theta(z)]^{-1}[\mathbf{I} - \Phi(z)]\mathbf{H}\mathbf{x}(t) + \mathbf{a}(t) \qquad (14)\end{aligned}$$

Note that the new measurement noise in Eq. (14) is white noise $\mathbf{a}(t)$, which is uncorrelated with the process noise $\mathbf{w}(t)$. Now let us define

$$\begin{aligned}\mathbf{g}(t) &= [\mathbf{I} - \Theta(z)]^{-1}[\mathbf{I} - \Phi(z)]\mathbf{H}\mathbf{x}(t) \\ &= [\mathbf{I} - \theta_1 z^{-1} - \cdots - \theta_q z^{-q}]^{-1}[\mathbf{I} - \phi_1 z^{-1} - \cdots - \phi_p z^{-p}]\mathbf{H}\mathbf{x}(t) \qquad (15)\end{aligned}$$

Multiplying $\mathbf{g}(t)$ by $[\mathbf{I} - \theta_1 z^{-1} - \cdots - \theta_q z^{-q}]$, we obtain

$$\begin{aligned}&\mathbf{g}(t) - \theta_1\mathbf{g}(t-1) - \theta_2\mathbf{g}(t-2) - \cdots - \theta_q\mathbf{g}(t-q) \\ &\qquad = \mathbf{H}\mathbf{x}(t) - \phi_1\mathbf{H}\mathbf{x}(t-1) - \cdots - \phi_p\mathbf{H}\mathbf{x}(t-p) \qquad (16)\end{aligned}$$

or

$$\begin{aligned}\mathbf{g}(t) &= \mathbf{H}\mathbf{x}(t) - \phi\mathbf{H}\mathbf{x}(t-1) - \cdots - \phi_p\mathbf{H}\mathbf{x}(t-p) + \theta_1\mathbf{g}(t-1) + \cdots + \theta_q\mathbf{g}(t-q) \\ &= \mathbf{H}^*\mathbf{x}^*(t) \qquad (17)\end{aligned}$$

So we can express $\mathbf{y}^*(t)$ as

$$\begin{aligned}\mathbf{y}^*(t) &= \mathbf{g}(t) + \mathbf{a}(t) \\ &= \mathbf{H}^*\mathbf{x}^*(t) + \mathbf{a}(t) \qquad (18)\end{aligned}$$

where

$$\mathbf{H}^* = [\mathbf{H} \mid -\phi_1\mathbf{H} \mid \cdots \mid -\phi_p\mathbf{H} \mid \theta_1 \mid \theta_2 \mid \cdots \mid \theta_q] \qquad (19)$$

$$[\mathbf{x}^*(t)]^T = [\mathbf{x}(t)^t \mid \mathbf{x}(t-1)^T \mid \cdots \mid \mathbf{x}(t-p)^T \mid \mathbf{g}(t-1)^T \mid \mathbf{g}(t-2)^t \mid \cdots \mid \mathbf{g}(t-q)^T] \qquad (20)$$

$\mathbf{H}^*$ is an $m \times [(p+1)n + qm]$ matrix and $\mathbf{x}^*(t)$ is a $[(p+1)n + qm] \times 1$ augmented vector. Note that $\mathbf{x}^*(t)$ consists of the current value $\mathbf{x}(t)$, the past p values of the $\mathbf{x}(t-k)$, $k = 1, 2, \ldots, p$, and the past q values of the supplementary state variable $\mathbf{g}(t-j)$, $j = 1, 2, \ldots, q$. Finally the system described by Eqs. (1), (2) and (4) after prewhitening and augmentation becomes
State Equation:

$$\mathbf{x}^*(t+1) = \mathbf{A}^*\mathbf{x}^*(t) + \mathbf{B}^*\mathbf{u}(t) + \mathbf{C}^*\mathbf{w}(t) \qquad (21)$$

Measurement Equation:

$$\mathbf{y}^*(t) = \mathbf{H}^*\mathbf{x}^*(t) + \mathbf{a}(t) \qquad (22)$$

where

$$
\mathbf{A}^* = \left[\begin{array}{ccccc|ccccccc}
\mathbf{A} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdot & \cdot & \mathbf{0} & \mathbf{0} \\
\mathbf{I} & \mathbf{0} & \mathbf{0} & & & \mathbf{0} & \mathbf{0} & \cdot & \cdot & \cdot & \cdot & \mathbf{0} \\
\mathbf{0} & \mathbf{I} & \mathbf{0} & & & \cdot & \cdot & & & & & \cdot \\
\mathbf{0} & \mathbf{0} & \mathbf{I} & & & \cdot & \cdot & & & & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot & \cdot & & \cdot & & & \cdot \\
\cdot & & & \mathbf{I} & & \cdot & \cdot & & & & \cdot & \cdot \\
\cdot & & & \cdot & & \mathbf{0} & \mathbf{0} & & & & & \cdot \\
\mathbf{0} & \cdot & \cdot & \cdot & \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdot & \cdot & \cdot & \mathbf{0} & \mathbf{0} \\
\hline
\mathbf{H} & -\phi_1\mathbf{H} & -\phi_2\mathbf{H} & \cdots & -\phi_{p-1}\mathbf{H} & -\phi_p\mathbf{H} & \theta_1 & \theta_2 & \theta_3 & \cdots & & \theta_q \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdot & \cdots & & \cdot \\
\cdot & \cdot & & \cdot & \cdot & \cdot & \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdot & & \vdots \\
\cdot & & & & & \cdot & \cdot & \cdot & \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdot \\
\cdot & & & \cdot & & \cdot & \cdot & & & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdot & \cdot & \mathbf{I} & \mathbf{0}
\end{array}\right] \tag{23}
$$

$$(\mathbf{B}^*)^T = [\mathbf{B}\,|\,\mathbf{0}\,|\,\mathbf{0}\,|\cdots|\,\mathbf{0}\,|\,\mathbf{0}\,|\cdots|\,\mathbf{0}] \tag{24}$$

and

$$(\mathbf{C}^*)^T = [\mathbf{I}\,|\,\mathbf{0}\,|\,\mathbf{0}\,|\cdots|\,\mathbf{0}\,|\,\mathbf{0}\,|\cdots|\,\mathbf{0}] \tag{25}$$

The dimensions of $\mathbf{A}^*$, $\mathbf{B}^*$, and $\mathbf{C}^*$ are $[(p+1)n+qm]\times[(p+1)n+qm]$, $[(p+1)n+qm]\times r$, and $[(p+1)n+qm]\times n$, respectively.

Equations (21) and (22) together are in the standard Kalman filter form and hence the discrete-time optimal filter recursive equations can be developed in a straightforward way as follows.

$$\hat{\mathbf{x}}^*(t^-) = \mathbf{A}^*\hat{\mathbf{x}}^*((t-1)^+) + \mathbf{B}^*\mathbf{u}(t-1) \tag{26}$$

$$\mathbf{P}^*(t^-) = \mathbf{A}^*\mathbf{P}^*((t-1)^+)(\mathbf{A}^*)^T + \mathbf{C}^*\mathbf{Q}(\mathbf{C}^*)^T \tag{27}$$

$$\mathbf{K}^*(t) = \mathbf{P}^*((t-1)^-)(\mathbf{H}^*)^T[\mathbf{H}^*\mathbf{P}^*((t-1)^-)(\mathbf{H}^*)^T + \mathbf{R}^*]^{-1} \tag{28}$$

$$\hat{\mathbf{x}}^*(t^+) = \hat{\mathbf{x}}^*(t^-) + \mathbf{K}^*(t)[\mathbf{y}^*(t) - \mathbf{H}^*\hat{\mathbf{x}}^*(t^-)] \tag{29}$$

$$\mathbf{P}^*(t^+) = (\mathbf{I} - \mathbf{K}^*(t)\mathbf{H}^*)\mathbf{P}^*((t-1)^-) \tag{30}$$

Note that the state space dimension is $(p+1)n+mq$ which depends on the order of the ARMA(p, q) model, but which is finite and linear function of p and q. Also, this approach does not require knowledge of infinitely many past values of the measurements as in Eq. (13). Also note that since the matrix $[\mathbf{H}^*\mathbf{P}^*((t-1)^-)(H^*)^T + \mathbf{R}^*]$ is invertible and $\mathbf{R}^* \neq \mathbf{0}$, so $\mathbf{P}^*(t^+)$ is not singular. Eqs. (26)–(30) do not increase the computational time tremendously because the critical dimension, namely, that of the matrix $[\mathbf{H}^*\mathbf{P}^*((t-1)^-)(\mathbf{H}^*)^T + \mathbf{R}^*]^{-1}$ remains unchanged.

For the case where $\mathbf{v}(t)$ follows the ARMA(1, 1) model, these quantities simplify to

$$\mathbf{x}^*(t) = \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{x}(t-1) \\ \mathbf{g}(t-1) \end{pmatrix} \quad \mathbf{A}^* = \begin{pmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{H} & -\phi_1\mathbf{H} & \theta_1 \end{pmatrix} \quad \mathbf{B}^* = \begin{pmatrix} \mathbf{B} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

$$\mathbf{C}^* = \begin{pmatrix} \mathbf{I} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \quad \text{and} \quad \mathbf{H}^* = [\mathbf{H} \mid -\phi_1\mathbf{H} \mid \theta_1] \tag{31}$$

MODEL WITH SERIALLY CORRELATED PROCESS AND MEASUREMENT NOISES

Let us now consider a more general case of Eqs. (1) and (2) in which the process noise as well as the measurement noise are serially correlated and follow the ARMA(p, q) models. For this case two steps are involved in prewhitening.

Step 1: We first prewhiten the measurement noise as given above in Eqs. (21) and (22) with the understanding that $\mathbf{w}(t)$ in Eq. (21) is not white.

Step 2: Since $\mathbf{w}(t)$ itself is serially correlated, we can then use the augmentation method (Stengel, 1986; Maybeck, 1979) to add it to the prewhitened state variables, $\mathbf{x}^*(t)$. Similar to the aforementioned procedure, we can define an augmented state vector $\mathbf{x}^{**}(t)$ such that

$$\begin{bmatrix} \mathbf{x}^*(t+1) \\ \mathbf{w}(t+1) \\ \mathbf{w}(t) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{w}(t-p+2) \\ \mathbf{w}(t-p+1) \end{bmatrix} = \left[\begin{array}{c|cccccc} \mathbf{A}^* & \mathbf{C}^* & \mathbf{0} & & \cdots & & \mathbf{0} \\ \hline \mathbf{0} & \phi_1 & \phi_2 & & \cdots & & \phi_p \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & & \cdot & \mathbf{0} & \mathbf{0} \\ \cdot & & & \cdot & & \cdot & \cdot \\ \cdot & & & \cdot\mathbf{I} & & \mathbf{0} & \cdot \\ \mathbf{0} & & & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & & \mathbf{I} & \mathbf{0} \end{array}\right] \begin{bmatrix} \mathbf{x}^*(t) \\ \mathbf{w}(t) \\ \mathbf{w}(t-1) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{w}(t-p+1) \\ \mathbf{w}(t-p) \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{B}^* \\ \mathbf{0} \\ \mathbf{0} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{u}(t) + \left[\begin{array}{cccc} \mathbf{0} & \cdot & \cdots & \mathbf{0} \\ \hline \mathbf{I} & -\theta_1 & \cdots & -\theta_q \\ \mathbf{0} & \cdot & \cdots & \mathbf{0} \\ \cdot & \cdot & & \cdot \\ \cdot & & & \cdot \\ \cdot & & \cdot & \cdot \\ \mathbf{0} & & & \cdot \\ \mathbf{0} & \cdot & \cdots & \mathbf{0} \end{array}\right] \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}(t-1) \\ \mathbf{n}(t-2) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{b}(t-q+1) \\ \mathbf{n}(t-q) \end{bmatrix}$$

or

$$\mathbf{x}^{**}(t+1) = \mathbf{A}^{**}\mathbf{x}^{**}(t) + \mathbf{B}^{**}\mathbf{u}(t) + \mathbf{C}^{**}\mathbf{n}^*(t) \tag{32}$$

and

$$\mathbf{y}^{**}(t) = [\mathbf{H}^* \mid \mathbf{0} \mid \mathbf{0} \mid \cdots \mid \mathbf{0}] \begin{bmatrix} \mathbf{x}^*(t) \\ \mathbf{w}(t) \\ \vdots \\ \mathbf{w}(t-p) \end{bmatrix} + \mathbf{a}(t)$$

or

$$\mathbf{y}^{**}(t) = \mathbf{H}^{**}\mathbf{x}^{**}(t) + \mathbf{a}(t) \tag{33}$$

Equations (32) and (33) together again are in the standard Kalman filter form in which all noises are white. Then the discrete-time optimal filter recursive equations can be obtained as before. Again the computational time does not increase tremendously since only a modest increase in the state space dimension is involved.

CONCLUSIONS

A general prewhitening procedure is proposed for dealing with a stochastic linear dynamic system with serially correlated process and measurement noises. This procedure should be used before applying the Kalman filter to obtain a minimum variance unbiased state estimate such that the resulting innovations form a white noise sequence. The standard techniques for gross error detection in dynamic systems can be applied thereafter. The prewhitening causes an increase in the state space dimension, but the increase is not serious enough to make the computations ill-conditioned or infeasible. The proposed approach is applicable to any ARMA model.

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REFERENCES

Bellingham, B., and Lees, F.P., "The Detection of Malfunction Using a Process Control Computer: A Kalman Filtering Technique for General Control Loops", *Trans. IChemE.*, **55**, 253 (1977).

Box, G.E.P., and Jenkins, G.M., *Time Series Analysis: Forecasting and Control,* Holden-Day: San Francisco (1976).

Bryson, A.E., and Henrikson, L.J., "Estimation Using Sampled Data Containing Sequentially Correlated Noise", *J. Spacecraft Rocket*, **5**, 662 (1968).

Bryson, A.E., and Ho, Y.C., *Applied optimal Control,* Massachusetts: Gin Blaisdell, pp. 403 (1969).

Kalman, R.E., "New Methods in Wiener Filtering Theory", *Proc. 1st Symp. on Engrg. Applications of Random Function Theory and Probability*, Bogdanoff, J.L., and Kozin, F., Eds.; New York: Wiley (1963).

Kao, C.S., Tamhane, A.C., and Mah, R.S.H., "Gross Error Detection in Serially Correlated Process Data", *I & EC Research*, **29**, 1004 (1990).

Kao, C.S., Tamhane, A.C., and Mah, R.S.H., "Gross Error Detection in Serially Correlated Process Data(II): Dynamic Systems", *I & EC Research*, **31**, 254 (1992).

Mah, R.S.H., *Chemical Process Structures and Information Flows*, Butterworth: Boston, (1990).

Maybeck, P.S., *Stochastic Models, Estimation, and Control, Vol. 1*, Academic Press Inc.; Florida, (1979).

Narasimhan, S., and Mah, R.S.H., "Generalized Likelihood Ratios for Gross Error Identification in Dynamics Processes", *AIChE J.*, **34**, 1321, (1988).

Stengel, R.F., *Stochastic Optimal Control: Theory and Application*, New York: Wiley, (1986).

Tamhane, A.C., and Mah, R.S.H., "Data Reconciliation and Gross Error Detection in Chemical Process Networks", *Technometrics*, **27**, 409 (1985).

Watanabe, K., and Himmelblau, D.M., "Instrument Fault Detection in Systems with Uncertainties", *Int. J. Systems Sci.*, **13**, 137 (1982).